**Mathematics in Context** is a comprehensive curriculum for the middle grades. It was developed in 1991 through 1997 in collaboration with the Wisconsin Center for Education Research, School of Education, University of Wisconsin-Madison and the Freudenthal Institute at the University of Utrecht, The Netherlands, with the support of the National Science Foundation Grant No. 9054928.

The revision of the curriculum was carried out in 2003 through 2005, with the support of the National Science Foundation Grant No. ESI 0137414.

---

**National Science Foundation**

Opinions expressed are those of the authors and not necessarily those of the Foundation.

---


---


All rights reserved.

No part of this work may be reproduced or utilized in any form or by any means, electronic or mechanical, including photocopying, recording or by any information storage or retrieval system, without permission in writing from the publisher.

International Standard Book Number 978-1-59339-944-3

Printed in the United States of America
The Mathematics in Context Development Team


The initial version of Made to Measure was developed by Anton Roodhardt and Jan Auke de Jong. It was adapted for use in American schools by Laura J. Brinker, James A. Middleton, and Aaron N. Simon.

Wisconsin Center for Education

Research Staff

Thomas A. Romberg
Director
Gail Burrill
Coordinator

Freudenthal Institute Staff

Jan de Lange
Director
Els Feijs
Coordinator

Project Staff

Jonathon Brendefur
Laura Brinker
James Browne
Jack Burrill
Rose Byrd
Peter Christiansen
Barbara Clarke
Doug Clarke
Beth R. Cole
Fae Dremock
Mary Ann Fix

Sarah Ailts
Beth R. Cole
Erin Hazlett
Teri Hedges
Karen Hoiberg
Carrie Johnson
Jean Krusi
Elaine McGrath

Mieke Abels
Nina Boswinkel
Frans van Galen
Koen Gravemeijer
Marja van den Heuvel-Panhuizen
Jan Auke de Jong
Vincent Jonker
Ronald Keijzer
Martin Kindt

Revision 2003–2005

The revised version of Made to Measure was developed Mieke Abels and Jan de Lange. It was adapted for use in American Schools by Margaret A. Pligge.

Wisconsin Center for Education

Research Staff

Thomas A. Romberg
Director
Gail Burrill
Editorial Coordinator

Freudenthal Institute Staff

Jan de Lange
Director
Truus Dekker
Coordinator

Project Staff

David C. Webb
Coordinator
Margaret A. Pligge
Editorial Coordinator

Sarah Ailts
Anne Park
Brynja Rappaport
Kathleen A. Steele
Ana C. Stephens
Candace Ulmer

Arthur Bakker
Peter Boon
Els Feijs
Dédé de Haan
Martin Kindt

Margaret R. Meyer

Huub Nilwik
Sonia Palha
Nanda Querelle
Martin van Reeuwijk

Nathalie Kuijpers

Jill Vettrus

Hyacinth Schuylers
Cover photo credits: (left) © Getty Images; (middle) © Kaz Chiba/PhotoDisc/ Getty Images; (right) © PhotoDisc/Getty Images

Illustrations
2, 14, 22 Holly Cooper-Olds; 23, 24 Christine McCabe/© Encyclopædia Britannica, Inc.; 28 Holly Cooper-Olds; 31 Christine McCabe/© Encyclopædia Britannica, Inc.; 32 Holly Cooper-Olds; 35 Christine McCabe/© Encyclopædia Britannica, Inc.; 37, 38 Holly Cooper-Olds

Photographs
1 (counter clockwise) © PhotoDisc/Getty Images; © PhotoDisc/Getty Images; © Ingram Publishing; © PhotoDisc/Getty Images; Sam Dudgeon/HRW Photo; © Corbis; 10 Victoria Smith/HRW; 17 Victoria Smith/HRW; 34 © EB Inc.
## Contents

Letter to the Student  vi

### Section A  Lengths

<table>
<thead>
<tr>
<th>Topic</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction</td>
<td>1</td>
</tr>
<tr>
<td>Historical Measures</td>
<td>2</td>
</tr>
<tr>
<td>Feet and Shoes</td>
<td>6</td>
</tr>
<tr>
<td>Body Length and Fathom</td>
<td>8</td>
</tr>
<tr>
<td>Other Measures for Length</td>
<td>9</td>
</tr>
<tr>
<td>Summary</td>
<td>10</td>
</tr>
<tr>
<td>Check Your Work</td>
<td>11</td>
</tr>
</tbody>
</table>

### Section B  Areas

<table>
<thead>
<tr>
<th>Topic</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>A Body’s Surface Area</td>
<td>12</td>
</tr>
<tr>
<td>Squares</td>
<td>13</td>
</tr>
<tr>
<td>Hands and Body</td>
<td>15</td>
</tr>
<tr>
<td>Surface Area by Formula</td>
<td>17</td>
</tr>
<tr>
<td>Height, Weight, and Area</td>
<td>18</td>
</tr>
<tr>
<td>Early Areas</td>
<td>19</td>
</tr>
<tr>
<td>Summary</td>
<td>20</td>
</tr>
<tr>
<td>Check Your Work</td>
<td>21</td>
</tr>
</tbody>
</table>

### Section C  Volumes

<table>
<thead>
<tr>
<th>Topic</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Volume of Your Heart</td>
<td>22</td>
</tr>
<tr>
<td>Solids</td>
<td>22</td>
</tr>
<tr>
<td>Liquids</td>
<td>26</td>
</tr>
<tr>
<td>The Volume of Your Body</td>
<td>27</td>
</tr>
<tr>
<td>Other Measures for Volume</td>
<td>28</td>
</tr>
<tr>
<td>Summary</td>
<td>30</td>
</tr>
<tr>
<td>Check Your Work</td>
<td>30</td>
</tr>
</tbody>
</table>

### Section D  Angles

<table>
<thead>
<tr>
<th>Topic</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Furniture</td>
<td>32</td>
</tr>
<tr>
<td>Summary</td>
<td>38</td>
</tr>
<tr>
<td>Check Your Work</td>
<td>38</td>
</tr>
</tbody>
</table>

### Additional Practice

<table>
<thead>
<tr>
<th>Topic</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>40</td>
</tr>
</tbody>
</table>

### Answers to Check Your Work

<table>
<thead>
<tr>
<th>Topic</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>43</td>
</tr>
</tbody>
</table>
Dear Student,

Welcome to the *Mathematics in Context* unit *Made to Measure*. This unit is all about measuring: measuring your feet, your thumb, your hands, and the angle made by your arm and your wrist. You will investigate how measuring units evolved. You will further investigate measurements for length, area, and volume. You might be amazed by what you can measure!

You will find that mathematics plays an important role in measurement. Every time you measure something, you might ask yourself:

- Will every person measuring this item get the same measurement that I did?
- Do all of these things have the same measurement?
- What other units of measure can I use?
- Are there other ways to measure these things?

Whenever you make a measurement in this unit, picture how big—or small or steep or short—that measurement is. When you can do this with all of the measurements in this unit, you are well on your way to becoming a mathematician!

Sincerely,

*The Mathematics in Context Development Team*
Introduction

People who design the objects you use every day have thought a lot about how big or how small those objects should be. Knowing the sizes of people’s arms, legs, and hands can be very useful when designing furniture, clothes, toys, windows, doors, and many other items.

1. Which body measures would be useful to know if you were designing the following items?
   a. doors                           d. pants
   b. school desks                    e. baby cribs
   c. shoes                          f. stairs

2. For what other objects would you need to know body measures?
You probably discovered that the lengths and heights of different body parts are important for designing many common objects. At one time, all measurements for length were related to the human body. Some of these units of measure include the thumb, hand span, foot, yard, pace, and fathom.

3. Match each of the units of measure listed in the paragraph above with its drawing below.

- **a.**
- **b.**
- **c.**
- **d.**
- **e.**
- **f.**
4. a. Which of the units of measure from problem 3 would you use to find the length of the nail shown here?

b. How long is this nail in the units of measure you chose for your answer to a?

5. a. Measure the length of your desk by using one or more of the units of measure from problem 3.

b. List the class results in a table. Did everyone find the same length? Why do you think this happened?

6. Reflect Name one advantage and one disadvantage of using your body to make measurements.

In Scotland during the Middle Ages, a unit of measure called the Scottish thumb was used. A Scottish thumb is the mean of the thumb widths of three men: a large man, an average-sized man, and a small man.

7. Why were three different-sized men used to determine the Scottish thumb?

In 1616, the Germans decided to create a unit of measure called the mean foot. To do this, they cut a piece of rope that was as long as the feet of 16 men.

8. a. How do you think the rope was used to find the length of the mean foot?

b. Which measurement is closer to the average person’s measurement: the German mean foot or the Scottish thumb? Explain your answer.

9. With the help of 16 classmates, find the length of the mean foot in your class by using the method described above.
10. Measure the following in centimeters (cm). List the results in a table.
   a. your thumb width       c. your foot length
   b. your hand span         d. your pace

11. Use the table you made in problem 10 to answer the questions about relationships between measures.
   a. How many thumb widths are in one hand span?
   b. How many thumb widths are in one foot?
   c. How many feet are in one pace?

You may add other units of measure and the relationships between them to your list.

12. What is the size of the “typical” thumb for your classmates? Explain your answer.

Think about what it must have been like when everyone used his or her own thumbs for measuring. Today, of course, everyone has standard systems of measurement.

A few countries, including the United States, still use the foot as a unit of measure, but the length of a foot no longer refers to the length of each person’s foot. A standard has been officially established for the length of a foot. Most countries use the metric system, which was adopted in France in 1795.

13. a. The foot is a part of the Imperial, or English, system of measurement. In the United States, we call this the customary system. List some other units of measure for length that are part of the customary system.
    b. In your notebook, write as many relationships between the units of measure in the customary system as you can.

14. a. List some units of measure for length that are part of the metric system.
    b. Write as many relationships between these units of measure as you can.

Since the United States officially uses the metric system, it is important for you to have a sense of how the metric and the customary systems relate. The next activity will help you find some simple relationships between the metric and customary systems.
Comparing Systems

Meters and Yards
- Use a meter stick to measure your classroom. Predict how the room dimensions would change if you measured with a yardstick.
- Use a yardstick to measure your classroom. Compare your prediction to your actual measurement.
- Find a conversion rule for meters and yards you can use when doing mental calculations.

Centimeters and Inches
- Use a centimeter ruler to measure a paper clip. Predict how the measurements would change if you measured using a ruler with inches.
- Use an inch ruler to measure the paper clip. Compare your prediction to your actual measurement.
- Find a conversion rule for centimeters and inches you can use when doing mental calculations.

Kilometers and Miles
Since athletes compete internationally, all distances are in meters (m) or kilometers (km). Today, many U.S. high school cross-country teams run 5-km races. Did you know that five kilometers is about three miles?
- Investigate how your school measures the running events. How long is the running track that your school uses?
- Name a location that is about one mile away from your school. Would a location that is about one kilometer away from your school be closer or farther?

15. Write as many relationships between units in the customary measurement system and the metric system as you can.
Feet and Shoes

In problem 10 c, you measured the length of your foot in centimeters. The length of your foot is different from your shoe size.

16. a. Do you think that there is a relationship between shoe size and foot length? Explain your answer.

b. Make a table that lists the foot length (in cm) and corresponding shoe size for each student in your class.

c. Graph the results. Put foot length in centimeters on the horizontal axis and shoe size on the vertical axis. What does your graph tell you about the relationship between foot length and shoe size?

Just as countries use different systems of measurement, they also have different systems for determining shoe sizes. For some shoes, you can find at least three different sizes:

- European size—usually a number between 33 and 47
- U.K. (United Kingdom) size—usually a number between 1 and 15
- U.S. size—usually a number between 1 and 15 (slightly larger than U.K. sizes)

The U.K. system of shoe sizes began in the seventh century.

Shoe sizes were measured with a standard thumb (now called an inch).

17. How many standard thumbs (or inches) long is your foot?

To get a more accurate measurement, the U.K. introduced a smaller unit of measure, the stitch. Three stitches are in one standard thumb.

18. How many stitches long is your foot?

In the U.K. system, shoe sizes are based on the number of stitches. The first 25 stitches are not counted in adult shoe sizes. Size 1 is, therefore, really 26 stitches, or \(8 \frac{2}{3}\) inches (in.).
19. a. Copy the table into your notebook and continue it to shoe size 8.

<table>
<thead>
<tr>
<th>Foot Length (in stitches)</th>
<th>Foot Length (in inches)</th>
<th>United Kingdom Shoe Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>26</td>
<td>8(\frac{2}{3})</td>
<td>1</td>
</tr>
<tr>
<td>27</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. Use your answer to problem 18 to find your U.K. shoe size in the table you made in part a. How does your U.K. shoe size compare with your U.S. shoe size?

c. Formulate a rule that helps you find someone’s foot length in inches if you know his or her U.K. shoe size. Write the rule in arrow language.

\[
\text{U.K. shoe size } \xrightarrow{?} \text{ ? } \xrightarrow{?} \text{ foot length (in inches)}
\]

20. Reflect After you finish problem 19, look back at your answers to problem 16. Would you change your answers now? Why or why not?

In problem 10 you measured your pace and your foot length.

21. Make a table of the pace and foot-length measurements of all of the students in your class. Use the results to determine whether the two measures are related. For example, you can find out if the person with the biggest foot also has the longest pace or if the person with the shortest foot has the shortest pace. Drawing a graph may be helpful.
Body Length and Fathom

The **fathom** is another unit of measure associated with the human body. You can measure a person’s fathom by having him or her stand tall and extend his or her arms out from both sides, horizontal to the ground. The fathom originated as the distance from the middle fingertip of one hand, to the middle fingertip of the other hand.

The picture is based on a famous drawing by Leonardo da Vinci. The girl more or less fits in a square.

22. Based on the picture on this page, what is the relationship between a person’s height and his or her fathom?

23. Measure your height and your fathom to decide how precisely you would fit into a square.
Other Measures for Length

There are many other ways to measure length. In Papua, New Guinea, for example, a local unit of distance is “a day’s travel.”

24. Why might a day’s travel make sense as a unit of distance?

In mountainous regions of New Guinea, walking distances are expressed in hours, not in kilometers or miles. Puli, a citizen of New Guinea, says, “It will take us two hours to cover the distance from the village to the lake in the mountains, but we save time on the return trip. The distance back will only take five quarters of an hour.”

25. a. Why do you think there are two travel times?

b. Is the distance in kilometers different for the two directions? Explain.

In the 14th century, the biggest trading empire in Africa was the Empire of Mali. Mansa Musa was one of its emperors. Sheik Uthman ed-Dukkali, a learned Egyptian who lived in Mali for 35 years, declared that Mali was “four months of travel long and three months wide.”

26. a. Use the map to estimate the length and width of the Empire of Mali in both customary and metric units of measure. (Note: 500 miles (mi) equals 800 km.)

b. What is the distance in miles or kilometers of “one month of travel”?

c. Based on your answer to part b, how do you think people in Mali traveled in the 14th century?
In this section, you learned many different ways to estimate and measure length. In the past, people used thumbs, feet, and arms to measure length, but each person often measured the same distance differently. Some people use units of time to measure distances. For example, Cedric takes one hour to hike around the nature trail.

Today, two standard systems of measurement exist. Most countries of the world use the **metric system**, in which length is measured in centimeters (cm), meters (m), and kilometers (km).

- 1 kilometer = 1000 meters
- 1 meter = 100 centimeters
- 1 centimeter = 10 millimeters

A few countries use the **customary, or Imperial, system**, in which length is measured in inches (in.), feet (ft), yards (yd), and miles (mi).

- 1 mile = 5,280 feet
- 1 yard = 3 feet
- 1 foot = 12 inches

Here are some relationships between both measuring systems. You may need to convert from one system to another.

- 1 mile is about 1.5 km (to be exact: 1 mi = 1.6 km)
- 1 yard is a little less than 1 m (to be exact: 1 yd = 0.9144 m)
- 1 foot is a little more than 30 cm (to be exact: 1 ft = 30.48 cm)
- 1 inch is about 2.5 cm (1 in. = 2.54 cm)
Check Your Work

1. Which unit of measurement would you use to measure the following lengths?
   a. the distance from your home to school
   b. the length of your classroom
   c. the thickness of a penny

2. List some distances expressed in units of time rather than units of length. Explain why using time is appropriate in each case.

3. Most rulers have markings for both the customary and metric systems.
   a. Another common metric unit is a decimeter (dm); 1 dm = 10 cm. Use a ruler to draw exactly one dm.
   b. Underneath your decimeter, use a ruler to draw 1 in.
   c. Estimate about how many inches are in 1 dm.
   d. Rewrite the list of metric measuring units in the Summary to include a decimeter.

Neville, who lives in Denmark, wrote to his friend in Texas. “Today my father and I went for a very long walk, about 16 km! I was very tired when we got home!”

4. Estimate how many miles Neville walked.

For Further Reflection

Make a list of your own reference points that have to do with length. For example, the distance from my house to our school is about three miles.
A body’s surface area is the amount of skin that covers a body. Sometimes it is important to know the surface area of a person’s body. For example, health care workers estimate the surface area of a burned patient to decide how much liquid the patient needs to replace lost fluids.

Body surface area is also important when caring for babies. Did you know that babies cool down faster than adults do? A person’s body cools down by sweating in relation to body surface area but warms up in relation to body mass. A baby’s skin area is very large in relation to his or her weight, so a baby cools down much faster than an adult. Babies can feel uncomfortably cold even when adults feel warm. So when you take care of a baby, don’t forget to monitor his or her skin temperature.

1. How might you measure your body’s surface area or amount of skin?

2. a. Estimate Ray’s body surface area in square centimeters. Ray is 157 cm tall, and his shoulder-to-shoulder width is 46 cm. Here is one square centimeter (cm²) you can use as a reference.
As you go through this section, you will estimate a person’s body surface area in many different ways. Here are measurement data for five different 7th graders.

<table>
<thead>
<tr>
<th>7th Grader</th>
<th>Age (yrs)</th>
<th>Height (cm)</th>
<th>Shoulder Width (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Joyce</td>
<td>12</td>
<td>135</td>
<td>38</td>
</tr>
<tr>
<td>Deon</td>
<td>13</td>
<td>147</td>
<td>46</td>
</tr>
<tr>
<td>Nora</td>
<td>12</td>
<td>151</td>
<td>43</td>
</tr>
<tr>
<td>Emmanuel</td>
<td>13</td>
<td>165</td>
<td>52</td>
</tr>
<tr>
<td>Luther</td>
<td>12</td>
<td>178</td>
<td>50</td>
</tr>
</tbody>
</table>

b. Estimate the body surface area of the 7th grader that is closest to your own height. Copy this table into your notebook to keep track of your estimations. Enter your method and estimate for the 7th grader you chose.

<table>
<thead>
<tr>
<th>7th Grader</th>
<th>Estimation Method</th>
<th>Estimate (cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Every time you make a new estimate, record your answer in your table.

**Squares**

In Section A, you investigated how well your body fits inside a square. Of course, not everyone’s square is the same size. For some people, it resembles a rectangle rather than a square.

You can use the area of a body’s square (or rectangle) to make another estimate of a person’s surface area. It might surprise you that three-fifths of the area of a body’s square is a good estimate of a body’s surface area.

3 Explain why \( \frac{3}{5} \) of a body’s square is a good estimate of a person’s surface area.
Skinning a Square

You will need a newspaper for this activity.

- Make a life-size body square out of newspaper. Use the data from one of the 7th graders on the previous page. You will probably need to tape some newspaper pages together to create the entire body square.

- Shade and measure $\frac{3}{5}$ of the square. Use square centimeters to calculate the area of the shaded part of your square. This is a new estimate for your 7th grader’s body surface area. Record the result in your Surface Area Estimation Table.

- How close do you think your estimate is? Cut up the shaded part of your square and try to piece together the front side of your 7th grader. Report how well your “skin” would cover your person. Save this person’s skin. You will need it later in the section.

There are other ways to find your body’s surface area. Timm Ulrichs, a German artist, did many artistic experiments that were quite mathematical. Mr. Ulrichs found his own surface area using small sticky squares. He took thousands of little sticky squares of foil, each exactly the size of one cm$^2$, and stuck them on his body until he was completely covered.

Since each piece of foil was 1 cm$^2$, Ulrichs was able to find the total surface area of his body. He placed all of the foil squares on graph paper in the shape of a large rectangle. He counted 18,360 squares and concluded that his body’s surface area was 18,360 cm$^2$.

4. Find some possible dimensions for the length and width of Timm Ulrichs’s rectangle. You don’t have to be exact.

5. a. Suppose Timm Ulrichs’s height is 180 cm. What is the area of a square with this height?

b. Is Timm Ulrichs’s body surface area of 18,360 cm$^2$ equal to $\frac{3}{5}$ of the square? Explain your answer.
Hands and Body

6. Trace your hand on cm-graph paper as shown. Estimate the area of your handprint in square centimeters.

Here is the height and width data for hand prints of the 7th graders.

<table>
<thead>
<tr>
<th>7th Grader</th>
<th>Age (yrs)</th>
<th>Hand Height (cm)</th>
<th>Hand Width (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Joyce</td>
<td>12</td>
<td>14</td>
<td>9</td>
</tr>
<tr>
<td>Deon</td>
<td>13</td>
<td>15</td>
<td>10</td>
</tr>
<tr>
<td>Nora</td>
<td>12</td>
<td>17</td>
<td>11</td>
</tr>
<tr>
<td>Emmanuel</td>
<td>13</td>
<td>18</td>
<td>14</td>
</tr>
<tr>
<td>Luther</td>
<td>12</td>
<td>21</td>
<td>13</td>
</tr>
</tbody>
</table>

You can use the area of your handprint to estimate your body’s surface area. Legend has it that it takes about 100 handprints to cover the body.

7. a. Use the handprint dimensions to sketch a handprint of one person.

b. Estimate the area of the handprint and the person’s body surface area.

c. Does the handprint method give the same estimate of the body’s surface area as Timm Ulrichs’s sticky squares method?

d. Record your new estimate in your Surface Area Estimation Table.
Burns can be very serious. The seriousness of a burn depends on how much of the body has been burned. To estimate the extent of a patient’s burns, health care workers use the “rule of nines.” This rule divides the body into eleven sections, each of which accounts for 9% of the total surface area, as shown in the picture.

8. a. Think of a way to measure as precisely as possible one of the “rule of nines” sections of the person you created in the activity on page 14. Measure the area of this section in square centimeters.

b. Use the result to calculate your person’s body surface area. Record your new estimate in your Surface Area Estimation Table.
There are both simple and complex formulas for finding the surface area of a person’s body. One simple way is to multiply your height by your thigh circumference (the length around your thigh) and double the answer. Here is a formula for this method.

\[
\text{height} \times \text{thigh circumference} \times 2 = \text{body surface area}
\]

9. Use the new formula and the measurement data for one 7th grader to estimate the person’s body surface area. Write your new estimate in your Surface Area Estimation Table.

<table>
<thead>
<tr>
<th>7th Grader</th>
<th>Age (yr)</th>
<th>Height (in cm)</th>
<th>Thigh Circumference (in cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Joyce</td>
<td>12</td>
<td>135</td>
<td>36</td>
</tr>
<tr>
<td>Deon</td>
<td>13</td>
<td>147</td>
<td>48</td>
</tr>
<tr>
<td>Nora</td>
<td>12</td>
<td>151</td>
<td>45</td>
</tr>
<tr>
<td>Emmanuel</td>
<td>13</td>
<td>165</td>
<td>54</td>
</tr>
<tr>
<td>Luther</td>
<td>12</td>
<td>178</td>
<td>48</td>
</tr>
</tbody>
</table>

The formula is based on the formula for the surface area of two cylinders.

10. Explain how you can model the body using two cylinders of equal height.

11. Reflect Would the formula work for babies? Explain why or why not.
Unfortunately, there is no easy formula to find out exactly how these three measures are related.

Instead, healthcare workers use a special chart called a **nomogram**.

First they find the patient’s height (in centimeters) in the left-hand column.

Next, they find the patient’s weight (in kilograms) in the far right hand column.

They connect these two points with a line and read the estimate for the patient’s surface area (in square centimeters) from the middle column. The estimate is where the line crosses the middle scale.

12. **a.** A line for one patient has already been drawn on this nomogram. What information about the patient is indicated by the nomogram?

Here is the weight data for the 7th graders.

**b.** To use the nomogram, the weight must be in kilograms. Use this ratio table to calculate the kilogram weight of each 7th grader.

**c.** Without drawing in your book, use the nomogram and a straightedge to estimate one 7th grader’s body surface area. Write your new estimate in your Surface Area Estimation Table.
On average, the surface area of an adult’s body is about two square meters (m²).

One square meter is exactly 10,000 cm² (100 cm × 100 cm).

13. Does the patient from problem 12a have an average body surface area? What about the 7th grader you have been working with?

**Early Areas**

In Section A, you learned how body measures were first used as measuring units for lengths. It wasn’t until later that measuring units became standard. Initially, area was measured using measuring units originating from the human body. Square feet, square thumbs, square hand span, and square fathoms, were used to measure area.

The English used the units **rood** and **acre**.

Area units in the metric system relate to units of length; square centimeters (cm²), square meters (m²), and square kilometers (km²) to name a few. Some Americans measure area with square inches (in.²), square feet (ft²), and square miles (mi²). To measure land area, they use an ancient unit called the **acre**. One acre is 43,560 ft² (1/640 mi² or 4047 m²). Unlike the other measuring units for area, the acre does not use the word “square” in its name. Parcels of land are often irregular in shape.

Suppose each of these pieces of land measures one acre.

14. a. What are the dimensions (in feet) of each acre?

   b. Make a scale drawing of a parcel of land that measures 10 acres.

The playing area of a football field is 100 yards long by 50 yards wide.

15. Use a calculation to show that a football field is about one acre.
Surface area is important for figuring out the amount of material needed to cover something. You use body surface area to determine the amount of fluid needed by burn victims.

Some metric units for measuring surface area are square centimeters and square meters. One square meter is the same as 10,000 square centimeters.

Some customary units for measuring surface area are square inches and square feet. One square foot is the same as 144 square inches.

An acre is an area unit for measuring land (1 acre = 43,560 ft²).

In this section, you learned about several different methods to find a person’s body’s surface area.

- \( \frac{3}{5} \) of the person’s square
- \( \text{height} \times \text{thigh circumference} \times 2 \)
- about 100 handprints
- a nomogram scale
Nicola is 1.58 m (or 158 cm) tall. She uses the method of \( \frac{3}{5} \) of a square to find her body’s surface area.

1. a. Calculate Nicola’s body’s surface area.
   b. Nicola’s answer to question 1a was 14,978.4 cm\(^2\). Why would 15,000 cm\(^2\) be a better estimate of her body’s surface area?
   c. Is Nicola’s surface area less than, more than, or equal to 2 m\(^2\)?

2. Trace your foot on graph paper and estimate the area of your footprint in square centimeters.

You have used this formula for a person’s surface area.

\[
\text{body surface area} = \frac{\text{height} \times \text{thigh circumference}}{2}
\]

This formula is based on the mathematical model of two cylinders.

There is another simple formula, based on the mathematical model of one cylinder.

\[
\text{body surface area} = \frac{\text{height} \times \text{hip circumference}}{2}
\]

3. a. Use the new formula to find the body surface area of an adult who is 1.85 m tall and has a hip circumference of 105 cm.
   b. Is the body surface area you found about average?

For Further Reflection

Write a paragraph or two comparing the different methods you used for estimating a person’s body surface area. Which of these methods do you think is most accurate? Why?
In the last two sections, you measured lengths and surface area. You can also measure, estimate, and calculate volume. Knowing the volume of a three-dimensional object is useful when you want to describe how much space it takes up.

Your heart is about the size of your two fists clenched together.

1. Estimate the volume of your heart.

**Solids**

You can use cubes to completely fill up a solid, a name for a 3-dimensional shape. To make a centimeter cube (cm³), draw a figure like the one shown here. The figure is called the net of a cube.

2. **a.** Use centimeter graph paper to draw a net of a centimeter cube. Cut it out and fold along the dotted lines. Tape the tabs to make one cubic centimeter (cm³).

   **b.** Does your class have enough centimeter cubes to fill up one cubic decimeter (dm³)? (Remember: 10 cm = 1 dm.)

   **c.** How many centimeter cubes do you need to completely fill one dm³?
Maria wants to find the volume of the box shown here. She started filling the box with the centimeter cubes.

Maria says, “I can easily find the volume of the box. Since the box measures 8 cm by 7 cm, I can fit 56 cubes on the bottom layer.”

5. Explain what else Maria has to do to find the volume of the box.

6. Find the volume of this container.

Find an empty tissue box.

3. a. Estimate how many centimeter cubes you would need to fill up the tissue box.

b. Cut off the top. How many centimeter cubes do you need to completely fill it? Explain how you found your answer.

If 32 centimeter cubes completely fill a box, then the volume of the box is 32 cm³. For larger objects, you would use larger measuring units. For example, you would measure the volume of your classroom in cubic decimeters or cubic meters (in the metric system) or in cubic feet or cubic yards (in the customary system).

4. a. For what other kinds of objects would you measure the volume in cubic centimeters? Cubic decimeters? Cubic meters?

b. Write some statements about how cubic centimeters, cubic decimeters, and cubic meters are related.

Maria wants to find the volume of the box shown here. She started filling the box with the centimeter cubes.

Maria says, “I can easily find the volume of the box. Since the box measures 8 cm by 7 cm, I can fit 56 cubes on the bottom layer.”

4. a. For what other kinds of objects would you measure the volume in cubic centimeters? Cubic decimeters? Cubic meters?

b. Write some statements about how cubic centimeters, cubic decimeters, and cubic meters are related.
The 1, 2, 3 Cheese Factory makes cheese cubes. They wrap each cube so that the cheese cubes look like number cubes. The edges of the cubes have a length of 2 cm.

7. a. How many of the one-cubic centimeter cubes would you need to fill the space taken by one cheese cube? (Hint: It may help to make a drawing.)

The 1, 2, 3 Cheese Factory packages the cheese cubes in a large plastic container, also the shape of a cube.

b. Would 100 cheese cubes exactly fill up this plastic cube? Explain why or why not.

The 1, 2, 3 Cheese Factory is designing a new plastic container that is 10 cm tall and holds exactly 60 cheese cubes.

c. What is the volume of this new plastic container?

d. List some possible dimensions for this new design. (Hint: It may help to make a drawing.)

Maria designed a 10-cm tall container with base dimensions 6 cm by 4 cm. She says, “To find the volume, I figured out the area of the base of the container and multiplied by 10 cm.”

8. a. Use Maria’s method to find the volume of her container.

b. How many cheese cubes would fit in her container?

9. Suppose that you want to determine the volume of your classroom.

a. Is it possible to use centimeter cubes (like the one you made on page 22) to find the volume of the classroom?

b. Can you use the area of one wall to find the volume of your classroom? Find the volume of your classroom. Be sure to use the right unit measurements.
10. Is it possible to use centimeter cubes (like the one you made on page 22) to find the volume of a soda can? Explain your reasoning.

Expressed as a formula, this was Maria’s strategy.

\[ \text{volume} = \text{area of the Base} \times \text{height} \]

11. Can this formula be used to find the volume of a stack of paper? A soda can?

The formula does not work for all three-dimensional objects. For example, you cannot find the volume of the pyramid by using this formula.

12. a. Give an example for which the formula does work and another example for which the formula does not work.

b. Reflect Why does the formula give the correct answer for some objects but not for others?

You can use the area of your handprint (Section B, problem 7) to estimate the volume of your hand.

13. a. Besides the area of your handprint, what other measure(s) do you need to estimate the volume of your hand?

b. Estimate the volume of your hand.
Liquids

The units used to measure liquids are different from the units used to measure solids. Pints, quarts, gallons, liters, and milliliters are all typical measures of liquid volume.

Here are some bottles whose contents are measured with units of liquid volume.

If you are estimating volumes, it may be helpful to know that:

- a regular soda can contains about $\frac{1}{3}$ liter.
- a cubic decimeter contains exactly one liter of liquid.

14. Estimate the volume (in liters) of the following objects.
   a. an orange juice container
   b. a large glass of water
   c. a small cup of juice
   d. a gallon of milk

Activity

Measuring the Volume of Your Hand

For this activity, you will need a can or beaker that measures in milliliters. It should be big enough so that you can put your hand in it.

Pour water into the can or beaker so that it is about half full. Measure and record the level of the water. Then, put a rubber band on your wrist, and put your entire hand in the water up to the bottom edge of the rubber band. You may want to make a fist. Be sure that your whole hand is underwater.

Measure and record the new water level. The difference between the old level and the new level is the volume of your hand in milliliters (ml).
You now have two estimates of the volume of your hand: one in cubic centimeters (from problem 13) and the other in milliliters. How are these two measurements related?

15. a. Make a table listing the two estimates of hand volumes for ten students.

b. Find a relationship between the two measurements—milliliters and cubic centimeters. Drawing a graph might be helpful.

<table>
<thead>
<tr>
<th>Milliliters</th>
<th>0</th>
<th>50</th>
<th>100</th>
<th>150</th>
<th>200</th>
<th>250</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cubic Centimeters</td>
<td>50</td>
<td>100</td>
<td>150</td>
<td>200</td>
<td>250</td>
<td></td>
</tr>
</tbody>
</table>

The Volume of Your Body

You could find the volume of your body in the same way that you found the volume of your hand, using liquid measures, but you would probably need a bathtub to do it!

Another way is to use solid units of measure and to think of your body as being made up of cubes.

16. Estimate your body’s volume by modeling your body with cubes as shown on the left. You will need to decide on a good unit of measure for the cubes.

To estimate the volume of your body, you can also use a block to model your body, as shown on the left.
The table above contains some information about Mindy when she was 10, 13, and 16 years old.

17. Use a block to model Mindy’s volume when she was 13 years old. Find the width and depth measurements of the block.

Other Measures for Volume

In the past, units of measure for volume were sometimes related to body sizes. It was more common, however, to use measurement tools like the cup as standards for measuring volume.

The ancient Egyptians had the first unit of measure for length—the cubit. The cubit is the distance from a person’s elbow to the tip of the extended middle finger.
The Egyptians used a standard length called the *royal cubit*, which is 52.4 cm, or 20.62 in.

In the ancient Egyptian system, the volume unit of measure is the *cubic cubit*.

18. a. Use your two arms to get a rough idea of how big a cubic cubit is.

b. About how many “royal” cubic cubits are there in a cubic meter?

The cubic fathom is a unit of measure used in the 1800s in Europe to measure volumes of firewood. (Remember that a fathom is the distance between your two outstretched arms.)

19. a. Estimate the amount of firewood that can fit into your cubic fathom.

b. What else can you measure with a cubic fathom? With a cubic cubit?

In the United States, firewood is measured in *cords*. A *face cord* is 4 ft high, 8 ft long, and 16 in. deep. A face cord usually fits in the back of a pickup truck. A *full cord* is 4 ft high, 8 ft long, and 4 ft deep.

20. How much more firewood is in a full cord than in a face cord?
Volume is important for measuring the sizes of such objects as a package, a drink, or even your hand.

In this section, you measured volume in two ways—with liquid units of measure (such as liters) and with solid units of measure (such as cubic centimeters).

You also measured the following:

- the volume of an object when you wanted to know how much space it takes up or how much it holds.
- the surface area or area of an object when you wanted to know what it takes to cover it.
- the length of an object when you wanted to know how long or how tall it is.

Check Your Work

1. When would it be useful to know the volume of an object?

2. Which unit measurement would you use for the following objects? (Choose from liters, cubic centimeters, and cubic meters.)
   a. a bottle of water
   b. the volume of water in a swimming pool
   c. the volume of a package containing math books
In a supermarket, you can buy small packages of fruit drinks in different sizes.

3. a. Could both packages show: Contains 0.2 liters? Why or why not?

b. Three packages of Frisca are packaged together and sold for $1.98. What is the price of one liter of Frisca?

For Further Reflection

Linda knows that one liter equals “one cubic something.” She doesn’t remember whether it is 1 cm$^3$, 1 dm$^3$, or 1 m$^3$. Explain to Linda which unit of measure equals one liter.
When designing furniture, cars, computers, and other items, it is sometimes necessary to know the angles that a person can make with his or her arms, ankles, wrists, and head.

1. For what objects might it be important to keep angles in mind?

Ergonomics is important for designing work environments. Designers use ergonomics to determine the placement of office equipment and the size and dimensions of furniture that will create safe, efficient working environments for people. Designers also consider ergonomics when creating new buses and trains for commuters, for example, by placing controls close to the driver.

2. Find the dictionary definition of ergonomics.

The first drawing shows the “arc of movement” of someone’s left hand. The picture shows how far to the left and right the wrist can bend.
You can measure the arc of movement of your own hands.

- Draw a small $x$ on the bottom of a sheet of paper.
- Lay your forearm flat, with your left hand and wrist on the sheet of paper. Put the middle of your wrist over the small $x$.
- Put a mark on the paper at the top of your middle finger.
- With your arm and hand flat on the table, bend your wrist to the right as far as you can without moving your arm. Put another mark on the paper at the top of your middle finger.
- Bend your wrist to the left as far as you can without moving your arm. Mark the location of the top of your middle finger.
- Draw lines from each of the three finger marks to the small $x$ (the middle of your wrist).

3. **a.** Use your compass card or protractor to measure the angles that your left hand can bend to the right and to the left.

   **b.** What do you think the angles will be for your right hand?

Measure the arc of movement of your right hand following the same instructions on page 32 and above.

4. **a.** Do you think the results of problem 3 will vary for different students in your class?

   **b.** Why might someone be interested in studying the motion of the wrist joint?

When typing on a computer keyboard, your hands should rest on the keys. In order to reach all of the keys, your hands must bend sideways. If you do a lot of typing, the position in which you hold your hands on the keyboard can cause physical discomfort in your hands, wrists, or forearms.
Studies on ergonomic keyboard design have found that most people prefer a *split keyboard*. A split keyboard is divided into two parts: the left part ends with the keys T-G-B, and the right part starts with keys Y-H-N.

On a split keyboard, the two parts make an angle of about 25°, that is, the lines drawn through T-G-B and Y-H-N make an angle of 25°. The distance between the two parts—or between the keys G and H—should be about 95 mm.

5. a. Make a drawing of a split keyboard. You do not have to draw all of the keys, only an outline of the two parts.

b. What is one advantage to using a split keyboard?

Another important angle in design involves your *line of sight*. If you look straight ahead, your line of sight follows a horizontal line. Your normal line of sight is typically 10° to 15° below the horizontal.

In the best ergonomic computer design, the screen should not be lower than the sight line of 15° below the horizontal.
Ellen uses a special device for her computer, which enables her to adjust the screen. This drawing is also shown on Student Activity Sheet 1.

Ellen sits at her chair in front of the computer screen. The horizontal line of sight from Ellen’s eye to the screen is at the midpoint of the screen. The distance from Ellen’s eye to the midpoint of the screen is 65 cm.

6. **a.** Use Student Activity Sheet 1 to find the exact place of Ellen’s eye while she is looking at her computer screen.

**b.** Is Ellen’s computer positioned according to the design recommendations?

You can also use ergonomics to design an easy chair. Some typical recommendations for the design of an easy chair follow.

- The seat should be tilted backward so that you will not slide off. A tilt of 14 to 24 degrees to the horizontal is recommended.

- The angle between the backrest and the seat should be from 105 to 110 degrees, and the angle between the backrest and the horizontal should be from 110 to 130 degrees.
Here are two different easy chairs.

7. Check to see if these chairs fit the recommendations on page 35.

8. Based on these recommendations, design your own easy chair. Draw a side view of your chair on an appropriate scale.

Shown here is the side view of an office chair.

9. a. Use Student Activity Sheet 1 to measure the angles the seat and the backrest make to the horizontal plane. Also measure the angle of the backrest to the seat. Draw extra lines if you want to.

b. Compare your findings to the recommendations above for easy chairs. What conclusion can you make?
The following text is taken from a book on ergonomics, *Fitting the Task to the Man*, by Etienne Grandjean.

“The arm can rotate through an angle of 250° about its axis...of which a half-circle (180°) lies in front of the body, and a further 70° or thereabouts, backwards.”

10. a. Read the text carefully. Write in your own words what the text means.

b. Check to see if your arm can make an angle of 70° backward. First, think of a way to measure this angle.

Math History

Challenged by the Egyptians!

It is sometimes said that measurement is the oldest appearance of mathematics. The name geometry literally means “measuring the earth.” Several pieces of papyrus have been found showing that over 5,000 years ago the Egyptians measured the area of the fields after the river Nile flooded. This may have been used to calculate the amount of taxes people had to pay. Many calculations and measurements were necessary to build the pyramids in Egypt.

The Egyptians also knew abstract facts about area and volume. The so-called Moscow Papyrus, illustrated here, shows how the volume of a truncated pyramid was calculated.

This is what the truncated pyramid looks like. The base is a $4 \times 4$ square and the top is a $2 \times 2$ square. The height is 6. Could you find the volume of this truncated pyramid? The answer is 58. Show how you found the answer.
In this section, you investigated the movement for both the right and left hand. You also explored how the placement of a computer screen and the angles between the seat and the backrest of an easy chair can affect your comfort.

Check Your Work

Frank is a small boy sitting behind a computer desk. The horizontal sight line from his eye to the screen ends at the midpoint of the screen.

1. a. Measure the angle between the two sight lines shown in the drawing.

b. Is the computer screen positioned according to the design recommendations?
(“In the best ergonomic computer design, the screen should not be lower than the sight line of 15° below the horizontal.”)

2. a. Spread your fingers to find the maximum angle between two of your fingers. Make a drawing to show your work.

b. Do you think all students in your class found approximately the same answer to problem 2a? Why or why not?

**For Further Reflection**

Complete the list of personal reference points you started in Section A. Write formulas you can use to convert commonly used measuring units. Adapt each formula to make it easy to do the conversions mentally.

<table>
<thead>
<tr>
<th>Customary to Metric</th>
<th>Metric to Customary</th>
</tr>
</thead>
<tbody>
<tr>
<td>miles → kilometers</td>
<td>kilometers → miles</td>
</tr>
<tr>
<td>inches → centimeters</td>
<td>centimeters → inches</td>
</tr>
<tr>
<td>feet → centimeters</td>
<td>centimeters → feet</td>
</tr>
<tr>
<td>pounds → kilograms</td>
<td>kilograms → pounds</td>
</tr>
</tbody>
</table>
1. List some body measures that would be useful to know if you were designing the following items.
   a. telephones
   b. children’s beds
   c. kitchen cabinets

2. List all of the units of (length) measure that you know that are related to the human body. Explain the meaning of each measure.

3. Which unit of measurement would you use to measure the following. (Note: Use the metric as well as the customary system.)
   a. the height of a door
   b. the length of a city block
   c. the length of a post-it-note

Here is the rule, expressed as an arrow string, to find someone’s foot length in inches if you know his or her U.K. shoe size:

\[
\text{U.K. size } \rightarrow +25 \rightarrow \text{_____} \rightarrow \div 3 \rightarrow \text{foot length (in inches)}
\]

4. a. Matthew wears size 5 in the U.K. system. What length (in inches) is his foot?
   b. Sondra measured her foot length, 9.5 in. Which shoe size (U.K. system) do you advise her to choose?

5. Which would you use to express the following distances—length or time? Explain.
   a. the distance from your home to school
   b. the distance from your city to New York City
   c. the distance from start to finish of a hiking trail
Section B Areas

1. Estimate the surface area of the following objects.
   a. a basketball   b. a book   c. a cereal box

2. For which object in problem 1 was it easiest to find the surface area? Why?

3. How many square centimeters are in one square meter?

The units for length in the metric system relate to each other.

- kilometers $\times 10$ → hectometers $\times 10$ → decameters $\times 10$ → meters
- $\times 10$ → decimeters $\times 10$ → centimeters $\times 10$ → millimeters

4. Make a similar arrow string showing the relationship between units for area in the metric system.

5. Enrique lives on a 15-acre farm.
   a. About how many football fields would cover his farm?
   b. What are possible dimensions for Enrique’s farm?

Section C Volumes

1. List all of the units of measure for volume that you know and explain how they relate to each other.

2. Find the volume in cubic centimeters of the following.

   a. Area of the base: 80 cm²
   b. 12 cm

   4 cm

   15 cm

   6 cm
One liter is one cubic decimeter (dm³).

3. a. What are possible dimensions (in inches) of a package with a volume of approximately one liter.
   
b. Use your answer for a to fill in the sentence.

    One liter is approximately __________ cubic inches.

4. Estimate the volume in liters of:
   
a. a bathtub
   b. a quart of milk
   c. a bottle of shampoo

---

**Section D Angles**

1. Draw a side view of a chair so that the angle between the seat and the backrest is 115° and the angle between the seat and the horizontal is 7°.

2. Below is a side view of a computer screen and a keyboard on a desk.
   
a. Measure the angle of the keyboard to the desk.
   
b. Measure the angle of the monitor to the horizontal plane of the desk.
1. Here are some possible units of measurement.
   a. Miles, kilometers, paces, minutes.
   b. Meters, decimeters, or feet. Many classrooms are about 8 m, or about 25 feet long.
   c. Millimeters. A penny has a thickness of about 1.5 mm.

Check with a classmate if your answers do not match any of these.

2. Here are some examples, but yours may be different.
   • Distance from home to school. It is important to know how much time it takes to go to school in order to be on time for school.
   • Walking in the mountains. In the mountains you may want to be back before dark, so it helps to know how long it will take. In general, walking speed is about 4 km or 2.5 miles per hour but much slower when you have to climb uphill or walk through sand.

3. a. Here is 1 dm.

   b. Here is an inch.

   c. A good estimate is about 4 in.; it is actually a little less than 4 in.

   d. 1 km = 1,000 m
      1 m = 10 dm or 100 cm
      1 dm = 10 cm

   Note that some measurements are not in the list: one hectometer (hm), one decameter (dam), and one millimeter (mm). The complete list follows:

   \[
   \begin{align*}
   1 \text{ km} &= 10 \text{ hm} & 1 \text{ m} &= 10 \text{ dm} \\
   1 \text{ hm} &= 10 \text{ dam} & 1 \text{ dm} &= 10 \text{ cm} \\
   1 \text{ dam} &= 10 \text{ m} & 1 \text{ cm} &= 10 \text{ mm}
   \end{align*}
   \]

4. About 10 miles. Using the information from problem 26: 500 miles is about 800 km, so 8 km is about 5 miles, and 16 km is about 10 miles.
1. a. About 15,000 cm².
   Rounding Nicola’s height up to 160 cm, Nicola’s personal square has an area of 25,600 cm². One-fifth is 5,120 cm² and three-fifths is 15,360 cm². Adjusting for rounding, a good estimate is 15,000 cm².

   b. Having a decimal in the answer is not appropriate here. This method is only an estimate, so 15,000 cm² is more reasonable for estimating.

   c. Nicola’s surface area is less than two square meters.
   Recall that 1 m² = 10,000 cm², since 1 m² is a 100 cm by 100 cm square and 100 cm × 100 cm = 10,000 cm²; 2 m² is double that amount, or 20,000 cm². Two square meters is well over Nicola’s surface area of 15,000 cm².

2. Estimates will likely be between 130–280 cm². Exchange your graph with a classmate to verify your estimate.

3. a. Using the new formula, the surface area is about 19,425 cm².
   Using hip circumference of 105 cm and a height of 185 cm, you calculate 185 cm × 105 cm = 19,425 cm².

   b. This is about average since the body surface area for an adult is about 2 m² and 19,425 cm² ≈ 1.9 m².

---

### Section C Volumes

1. Here is one response. Yours may be different.
   You might need to know an object’s volume for packing, shipping, or pricing.

2. a. a bottle of water: 1 liter.

   b. the volume of the water in a swimming pool: 900 m³.

   c. the volume of a package of math books: 3,000 cm³.
3. a. Yes, both packages can show, “contains 0.2 liters.” One liter is the same as 1,000 cm³ (1 dm³), so 0.2 liters is about 200 cm³. The shorter package has a volume of 202.5 cm³ (7.5 cm × 4.5 cm × 6 cm). The taller package has a volume of 216 cm³ (4.5 cm × 4 cm × 12 cm). The volume of both packages needs to be a little more than 200 cm³, so the juice will not spill out of the container too easily.

b. One liter costs $3.30. Sample explanation:

Three packages contain 0.6 liters (3 × 0.2 liters). Use a ratio table to find the price of one liter.

<table>
<thead>
<tr>
<th>Volume (in liters)</th>
<th>0.6</th>
<th>0.3</th>
<th>0.1</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price (in dollars)</td>
<td>1.98</td>
<td>0.99</td>
<td>0.33</td>
<td>3.30</td>
</tr>
</tbody>
</table>

Section D Angles

1. a. The angle measures about 30°.
   b. The angle below the horizontal sight line is half of 30° or 15°. The angle requirements are satisfied, but the screen is too high!

2. a. Your angle probably measures between 30° and 40°.
   b. No, probably not. Some students have a wider angle between their fingers than others. Compare your results with those of classmates.